

# UNIT – I INDEFINITE INTEGRATION

## 1. Basic Concept

Indefinite Integration is the **reverse process of differentiation**.

If:

$$d[F(x)] = f(x) \frac{d}{dx} [F(x)] = f(x) dx \quad d[F(x)] = f(x)$$

Then:

$$\int f(x) dx = F(x) + C \quad \int f(x) dx = F(x) + C$$

 **C is the constant of integration**


Because differentiation removes constants, we must add it back.


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## 2. Fundamental Rules

### (1) Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$


 Condition:  $n \neq -1$

 Examples:

- $\int x^2 dx = \frac{x^3}{3} + C$
  - $\int x^5 dx = \frac{x^6}{6} + C$
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### (2) Constant Rule

$$\int a dx = ax + C \quad \int a dx = ax + C$$

 Example:

$$\int 6 dx = 6x + C \quad \int 6 dx = 6x + C$$

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### (3) Constant Multiple Rule

$$\int a f(x) dx = a \int f(x) dx \quad \int a f(x) dx = a \int f(x) dx$$

 Example:

$$\int 4x^3 dx = x^4 + C \quad \int 4x^3 dx = x^4 + C$$

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### (4) Sum/Difference Rule

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx \quad \int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

✓ Example:

$$\int (x^2 + 3x) dx = \frac{x^3}{3} + \frac{3x^2}{2} + C$$

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### (5) Special Rule (Very Important)

$$\int \frac{1}{x} dx = \ln |x| + C$$

👉 Power rule does NOT apply here.

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## 3. Standard Integrals

### ◆ Exponential Functions

- $\int e^x dx = e^x + C$
  - $\int a^x dx = \frac{a^x}{\ln a} + C$
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### ◆ Trigonometric Functions

Function	Integral
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$\csc^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\csc x \cot x$	$-\csc x$

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## 4. Methods of Integration

### (A) Direct Integration

👉 Apply formula directly

Example:

$$\int (2x + 5) dx = x^2 + 5x + C$$

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## ◆ (B) Substitution Method (Basic Idea)

👉 Used when function is inside another function

Example:

$$\int (2x)(x^2+1)^3 dx \quad \int (2x)(x^2+1)^3 dx \quad \int (2x)(x^2+1)^3 dx$$

Let:

$$t = x^2 + 1 \quad dt = 2x dx$$

Then:

Integral becomes simple in terms of t.

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## ◆ (C) Partial Fractions (Very Important 🔥)

**Condition:**

👉 Degree of numerator < degree of denominator

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**Steps:**

1. Break into simpler fractions
  2. Find constants
  3. Integrate
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✓ **Example:**

$$\int \frac{3x+5}{(x+1)(x+2)} dx \quad \int \frac{3x+5}{(x+1)(x+2)} dx \quad \int \frac{3x+5}{(x+1)(x+2)} dx$$

Step 1:

$$\frac{3x+5}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow 3x+5 = A(x+2) + B(x+1)$$

Step 2:

$$3x+5 = A(x+2) + B(x+1) \Rightarrow 3x+5 = Ax+2A+Bx+B$$

Solve:

👉  $A=2, B=1$

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Step 3:





$$\int \frac{2}{x+1} + \frac{1}{x+2} dx = 2 \ln|x+1| + \ln|x+2| + C$$

Final Answer:

$$2 \ln|x+1| + \ln|x+2| + C$$

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## 5. Common Mistakes

-  Forgetting +C
  -  Using power rule for  $1/x$
  -  Writing  $\ln x$  instead of  $\ln|x|$
  -  Errors in partial fractions
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## 6. Quick Revision

- Integration = reverse of differentiation
- Always add +C
- Power rule is most used
- $1/x \rightarrow \ln|x|$  ;  $1/x \rightarrow \ln|x|$
- Memorize standard integrals
- Practice partial fractions

# **UNIT – II: DEFINITE INTEGRATION**

## **1. What is Definite Integration?**

Definite integration gives a **fixed numerical value**.

$$\int_a^b f(x) dx = F(b) - F(a)$$

👉 Key idea:

- First find **antiderivative**  $F(x)$
- Then apply limits

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## **2. Step-by-Step Method**

✓ **Example:**

$$\int_1^3 3x^2 dx$$

**Step 1: Integrate**

$$= [x^3] = \left[ \frac{x^3}{3} \right] = [x^3]$$

**Step 2: Apply limits**

$$= 27 - 1 = 26 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3} = 327 - 31 = 326$$

👉 This is your **final answer (number)**

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## **3. Important Rules (Must Know)**

### ◆ **Constant Rule**

$$\int_a^b k dx = k(b-a)$$

### ◆ **Sum Rule**

$$\int_a^b (f+g) dx = \int_a^b f dx + \int_a^b g dx$$

### ◆ Reversing Limits

$$\int_a^b f(x) dx = -\int_b^a f(x) dx \quad \int_a^a f(x) dx = -\int_a^a f(x) dx = 0$$

👉 Very common exam trick

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### ◆ Same Limits

$$\int_a^a f(x) dx = 0 \quad \int_a^a f(x) dx = 0$$

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## 📌 4. Properties (Shortcut Tricks 🔥)

### ◆ Odd Function

If:

$$f(-x) = -f(x) \quad f(-x) = -f(x)$$

Then:

$$\int_{-a}^a f(x) dx = 0 \quad \int_{-a}^a f(x) dx = 0$$

✓ Example:

$$\int_{-2}^2 x^3 dx = 0 \quad \int_{-2}^2 x^3 dx = 0$$

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### ◆ Even Function

If:

$$f(-x) = f(x) \quad f(-x) = f(x)$$

Then:

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \quad \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

✓ Example:

$$\int_{-2}^2 x^2 dx = 2 \int_0^2 x^2 dx \quad \int_{-2}^2 x^2 dx = 2 \int_0^2 x^2 dx$$

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### ◆ Splitting Property

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$
$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

👉 Useful when limits are tricky

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## 📌 5. Geometrical Meaning (Important Concept)

👉 Definite integral represents **area under curve**

$$\text{Area} = \int_a^b f(x) dx$$

### Important:

- Above x-axis → positive
- Below x-axis → negative

⚠️ If question asks **total area** → take absolute value

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## 📌 6. Numerical Integration

Used when integration is difficult.

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### ◆ (1) Trapezoidal Rule

$$\int_a^b f(x) dx \approx \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + \dots)]$$

👉 Simple but less accurate

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### ◆ (2) Simpson's 1/3 Rule (VERY IMPORTANT)

$$\int_a^b f(x) dx \approx \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots)]$$

👉 Condition: **n must be even**

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✓ **Example:**

x: 0, 1, 2

y: 1, 3, 5

h = 1

$$= \frac{1}{3} [1 + 5 + 4(3)] = \frac{1}{3} [1 + 5 + 12] = \frac{18}{3} = 6$$

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## 7. Common Exam Mistakes

- ✗ Forgetting to apply limits
  - ✗ Sign mistake in upper/lower limit
  - ✗ Ignoring odd/even property
  - ✗ Using Simpson when n is odd
  - ✗ Not taking absolute value in area
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## 8. Important Question Types

👉 Expect these in exam:

1. Direct definite integrals
  2. Using properties (odd/even)
  3. Area under curve
  4. Numerical methods (Simpson's rule)
  5. Changing limits
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## 9. Quick Revision (1-minute)

- Formula:  $F(b) - F(a)$
- Answer is always a number
- Odd  $\rightarrow 0$
- Even  $\rightarrow$  double
- Area concept important
- Simpson's rule  $\rightarrow$  n even

# UNIT – III: DIFFERENTIAL EQUATIONS

## 1. What is a Differential Equation?

A differential equation is an equation that contains:

- a dependent variable ( $y$ )
- an independent variable ( $x$ )
- derivatives ( $dy/dx$ )

✓ Example:

$$y \frac{dy}{dx} + y = x$$

👉 Used in:

- physics (motion)
  - engineering
  - population growth, decay, etc.
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## 2. Order and Degree

### ◆ Order

👉 Highest order of derivative

Examples:

- $\frac{dy}{dx} + y = 0$  → Order = 1
  - $\frac{d^2y}{dx^2} + y = 0$  → Order = 2
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### ◆ Degree

👉 Power of highest order derivative

Examples:

- $(\frac{d^2y}{dx^2})^2 + y = 0$  → Degree = 2

- $d^2y/dx^2 + y = 0$  → Degree = 1
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### 3. Formation of Differential Equation

👉 We form DE by eliminating constants

✓ **Example:**

Given:

$$y = mx + c$$

Differentiate:

$$\frac{dy}{dx} = m$$

Substitute:

$$y = x \frac{dy}{dx} + c$$

👉 This is the required DE

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### 4. Types of Differential Equations

- ◆ (1) Linear Differential Equation (Most Important 🔥)

Standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$


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### 5. Method to Solve Linear DE

👉 This is the **main exam method**

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**Step 1: Identify P(x) and Q(x)**

Example:

$$y \frac{dy}{dx} + y = x \frac{dy}{dx} + y = x$$

Here:

- $P(x) = 1$
- $Q(x) = x$

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### Step 2: Find Integrating Factor (I.F.)

$$I.F. = e^{\int P(x) dx} = e^{\int 1 dx} = e^x$$

👉 Here:

$$I.F. = e^{\int 1 dx} = e^x$$

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### Step 3: Multiply whole equation

$$e^x y \frac{dy}{dx} + e^x y = x e^x \frac{dy}{dx} + e^x y = x e^x$$

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### Step 4: Convert into derivative form

$$d(y e^x) = x e^x dx$$

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### Step 5: Integrate both sides

$$\int d(y e^x) = \int x e^x dx$$

👉 Solve RHS using integration by parts:

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

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### Step 6: Final answer

$$y e^x = x e^x - e^x + C$$

Divide by  $e^x$ :

$$y = x - 1 + C e^{-x}$$

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## 6. Exact Differential Equation

Form:

$$M(x,y)dx + N(x,y)dy = 0$$

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**Condition for Exactness:**

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

👉 If true → equation is exact

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## 7. Method to Solve Exact DE

**Step 1:**

Check condition

**Step 2:**

Integrate  $M$  w.r.t  $x$

**Step 3:**

Add function of  $y$

**Step 4:**

Differentiate w.r.t  $y$

**Step 5:**

Compare with  $N$  and find function

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## 8. Important Question Types

👉 Expect in exam:

1. Solve linear DE using I.F.
  2. Find integrating factor
  3. Check exactness
  4. Solve exact equation
  5. Form differential equation
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## 9. Common Mistakes

- ✗ Wrong integrating factor
  - ✗ Not multiplying whole equation
  - ✗ Mistakes in integration
  - ✗ Forgetting constant C
  - ✗ Not checking exact condition properly
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## 10. Quick Revision

- Form:  $dy/dx + Py = Q$
- I.F. =  $e^{\int P dx}$
- Multiply  $\rightarrow$  convert to derivative
- Integrate both sides
- Exact: check  $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

# UNIT – IV TRANSFORM

## TECHNIQUES

This unit is **concept + formula based**, and once you remember patterns, it becomes very scoring.

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### 1. What is Laplace Transform?

It converts a function from **time domain (t)** to **s-domain**.

👉 Why?

Because differential equations become **easy algebraic equations**.

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### 2. Definition

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \quad \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

👉 You don't usually solve this integral in exams

👉 You **use standard formulas directly**

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### 3. Standard Laplace Transforms (Must Memorize 🔥)

Function $f(t)$	Laplace Transform
1	$1/s$
t	$1/s^2$
$t^n$	$n! / s^{n+1}$
$e^{at}$	$1/(s-a)$
$\sin(at)$	$a / (s^2 + a^2)$

$\cos(at)$

$$\frac{s}{(s^2+a^2)} \frac{s}{(s^2+a^2)} \frac{s}{(s^2+a^2)}$$

👉 These come directly in exams.

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## 4. Important Properties

### ◆ (1) Linearity

$$\mathcal{L}\{af(t)+bg(t)\}=a\mathcal{L}\{f(t)\}+b\mathcal{L}\{g(t)\}$$
$$\mathcal{L}\{a f(t) + b g(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

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### ◆ (2) First Shifting Property

$$\mathcal{L}\{e^{at}f(t)\}=F(s-a)$$
$$\mathcal{L}\{e^{\{at\}} f(t)\} = F(s - a)$$

👉 Very important trick question

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### ◆ (3) Differentiation Property

$$\mathcal{L}\{f'(t)\}=sF(s)-f(0)$$
$$\mathcal{L}\{f'(t)\}=sF(s)-f(0)$$

👉 Used in solving differential equations

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## 5. Inverse Laplace Transform

👉 Converts back from **s-domain** → **time domain**

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### Standard Inverse Results

Function	Inverse
$1/s$	1
$1/s^2$	t

$$\frac{1}{s-a} \rightarrow e^{at}$$

$$\frac{s}{s^2+a^2} \rightarrow \cos(at)$$

$$\frac{a}{s^2+a^2} \rightarrow \sin(at)$$

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## 6. How to Solve Problems

### Example:

Find:

$$\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$

👉 Compare with standard:

$$\frac{1}{s-a} \rightarrow e^{at}$$

✓ Answer:

$$e^{2t}$$

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## 7. Application (Important Theory Question)

Laplace Transform is used to:

- Solve differential equations
  - Analyze electrical circuits
  - Study control systems
  - Engineering problems
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## 8. Common Mistakes

- ✗ Forgetting formulas
- ✗ Confusing sin and cos forms
- ✗ Sign mistake in  $s-a$
- ✗ Not applying shifting property correctly

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## 9. Quick Revision

- Learn standard transforms
- Practice inverse transforms
- Use properties smartly
- Don't try to derive—just apply

# UNIT – V: STATISTICS

## 1. What is Statistics?

Statistics deals with:

- collecting data
- organizing data
- analyzing data
- interpreting results

 Used in:

- business
  - economics
  - education
  - research
- 



## 5.1 Measures of Central Tendency


 These give a **single value representing the whole data**

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### ◆ (1) Mean (Arithmetic Mean)

**Formula:**

$$\text{Mean} = \frac{\sum x}{n} \quad \text{Mean} = \frac{\sum fx}{\sum f}$$

 For grouped data:

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

✓ **Example:**

Data: 2, 4, 6

$$\text{Mean} = (2+4+6)/3 = 4$$

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### ✓ Advantages:

- Uses all data
- Easy to calculate

### ✗ Disadvantages:

- Affected by extreme values (outliers)
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## ◆ (2) Median

👉 Middle value after arranging data

### Cases:

- If n is odd:

Median =  $(n+1)/2$ th term

- If n is even:

Median =  $\frac{n/2 \text{th term} + (n/2+1) \text{th term}}{2}$

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### For Grouped Data:

Median =  $l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$

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### ✓ Advantage:

- Not affected by extreme values
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## ◆ (3) Mode

👉 Most frequent value

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### For Grouped Data:

$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$   
 $\text{hMode} = l + (2f_1 - f_0 - f_2) \times h$

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### ✓ Advantage:

- Easy to understand
  - Useful in real-life data
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## 5.2 Measures of Dispersion

👉 Show how much data is spread

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### ◆ (1) Mean Deviation

$\text{Mean Deviation} = \frac{\sum |x - A|}{n}$   
 $\text{Deviation} = n \sum |x - A|$

👉 For grouped:

$= \frac{\sum f |x - A|}{\sum f} = \frac{\sum f |x - A|}{\sum f}$

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### ◆ (2) Standard Deviation (VERY IMPORTANT 🔥)

$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$   
 $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

👉 Shortcut formula:

$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - (\text{mean})^2}$   
 $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - (\text{mean})^2}$

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### ✓ Interpretation:

- Small SD → data close to mean
- Large SD → data spread out






## 5.3 Correlation

👉 Measures relationship between two variables

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### ◆ Types:

- Positive correlation  (both increase)
  - Negative correlation  (one increases, other decreases)
  - Zero correlation  (no relation)
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### ◆ Rank Correlation (Spearman's)

#### Formula:

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

👉 Where:

- $d$  = difference in ranks
  - $n$  = number of observations
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#### ✓ Steps:

1. Assign ranks
  2. Find difference ( $d$ )
  3. Find  $d^2$
  4. Apply formula
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#### ✓ Interpretation:

- $r = +1$  → perfect positive
- $r = -1$  → perfect negative
- $r = 0$  → no relation

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## Applications of Statistics

- Study performance analysis
- Business decisions
- Market trends
- Economics

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## Common Mistakes

- ✗ Wrong formula selection
- ✗ Mistakes in grouped data calculation
- ✗ Ignoring frequency (f)
- ✗ Calculation errors in SD
- ✗ Rank mistakes in correlation

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## Quick Revision

- Mean = average
- Median = middle value
- Mode = most frequent
- SD = spread of data
- Correlation = relationship